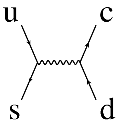
Combining Helicity Amplitudes With Resummation Using SCET

Wouter Waalewijn
UCSD



Fermliab Theory Seminar - April 4, 2013

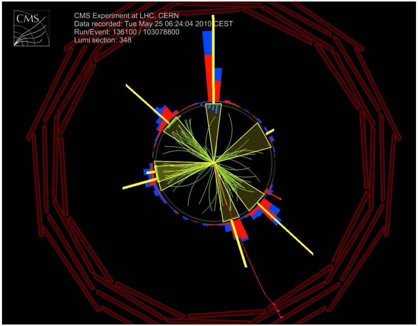
In collaboration with Iain Stewart and Frank Tackmann

Outline

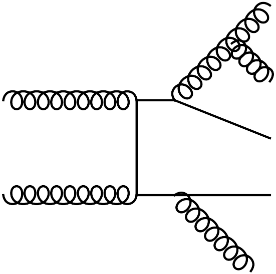
Introduction

Calculations and the LHC

Experiment:



Theory:



- Fixed-order calculations suitable for inclusive measurements
- Jet selection cuts require more detailed description of final state

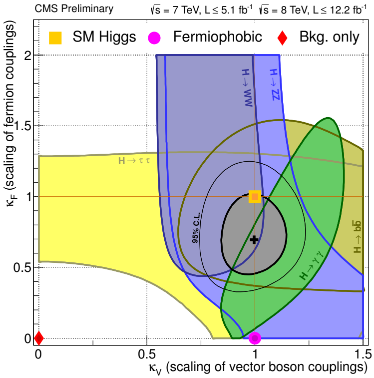
Example: Higgs Couplings

Assume universal rescaling of:

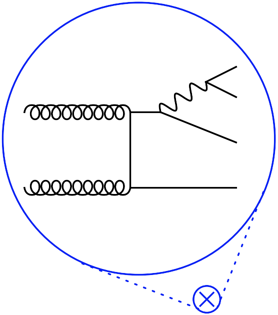
- vector couplings κ_V
- fermion couplings κ_F

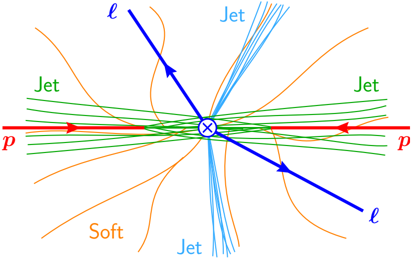
Several channels involve jet selection cuts:

- $H \to WW \to \ell\nu\ell\nu$ requires jet veto to remove $t\bar{t} \to WWb\bar{b}$ background
- $H \to \tau\tau$ most sensitive to vector boson fusion (2 jet selection)



Exclusive Measurements





- Hard process is accompanied by
  - Collinear ISR and collinear FSR
  - Soft radiation (underlying event)
- Hadronization effects
- Physics at multiple scales: $p_T^{\text{jet}} \gg p_T^{\text{veto}} \gg \Lambda_{\text{QCD}}, \ldots$
- May be described/modeled by Monte Carlo [MC@NLO, POWHEG]

Multi-Scale Cross Section

Cross section contains logarithms: $L = \ln(p^{\text{cut}}/\sqrt{\hat{s}})$

- Important when cut on hadronic final state $p^{\text{cut}} \ll$ hard scale $\sqrt{\hat{s}}$

Terms in the cross section

$$\begin{aligned}\sigma = \sigma_0\{1 &+ \alpha_s[c_{12}L^2 + c_{11}L + c_{10} + n_1(p^{\text{cut}})] \\ &+ \alpha_s^2[c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20} + n_2(p^{\text{cut}})] \\ &+ \alpha_s^3[c_{36}L^6 + c_{35}L^5 + c_{34}L^4 + c_{33}L^3 + c_{32}L^2 + \dots] \\ &+ \quad \vdots \quad + \quad \vdots \quad + \quad \vdots \quad + \quad \vdots + \quad \ddots\}\end{aligned}$$

Different calculations:

- Fixed order: LO, NLO, NNLO, ...

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- Fixed order: LO, NLO, NNLO, ...
- Monte Carlo: Parton-shower, MC@NLO

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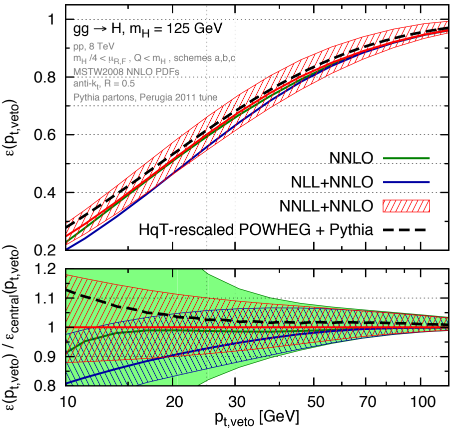
Different calculations:

- Fixed order: LO, NLO, NNLO, ...
- Monte Carlo: Parton-shower, MC@NLO
- Resummed: LL, NLL, NNLL, ...

SCET focuses on the numerically important singular contributions

- Nonsingular $n_i(p^{\text{cut}})$ from fixed-order calculation

Example: Higgs + 0 jets

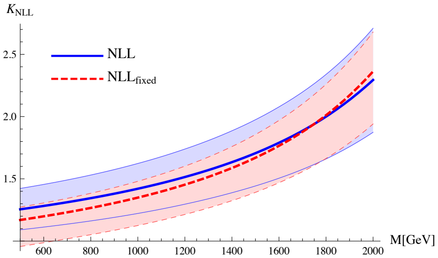


[Banfi, Monni, Salam, Zanderighi]

- Resummation improves prediction for $p_{T,\text{veto}} \ll m_H$
- Cross section for $p_{T,\text{veto}} \sim m_H$ given by fixed-order

Other Resummation Examples

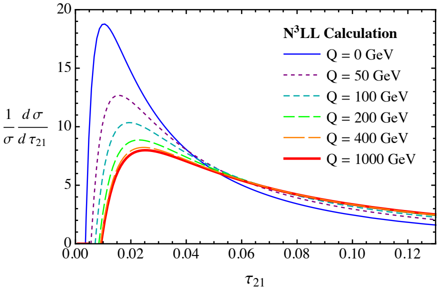
Gluino production



[Falgari, Schwinn, Wever]

- K-factor enhancement from threshold resummation
- $L = \ln(1 - 4M_{\tilde{g}}^2/\hat{s})$

Subjettiness τ_{21}

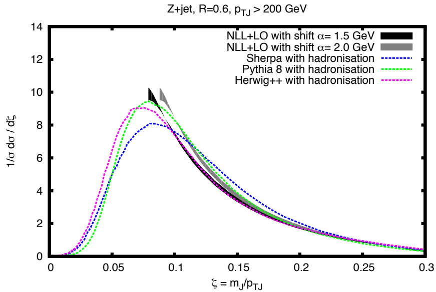


[Feige, Schwartz, Stewart, Thaler]

- Two-subjets in boosted $Z \to q\bar{q}$ jet of energy Q
- $L = \ln \tau_{21}$

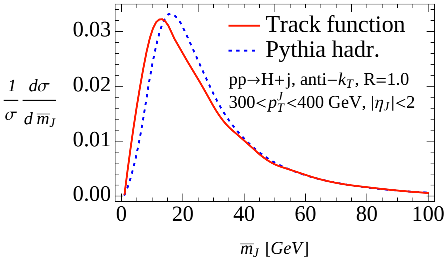
Hadronization Effects

Jet mass in $pp \to Z+$jet



[Dasgupta, Khelifa-Kerfa, Marzani, Spannowsky]

Track-based jet mass



[Chang, Procura, Thaler, WW]

- Hadronization shifts: $m_J^2 \to m_J^2 - \alpha\, R\, p_{TJ}$ with $\alpha \sim 2$ GeV
- Conversion to charged particles described by track functions

Goal

- Powerful methods for calculating helicity amplitudes analytically and numerically exist. Used in various programs [MCFM, Rocket, BlackHat, ...]
- We incorporate helicity amplitudes in Soft-Collinear Effective Theory using a helicity operator basis compatible with
  - Standard color structures
  - Discrete symmetries
  - Crossing symmetry
- Our paper will illustrate ease of use with explicit LO and NLO results for:
  - $pp \to W/Z/\gamma + 0, 1, 2$ jets
  - $pp \to H + 0, 1, 2$ jets
  - $pp \to 2, 3$ jets

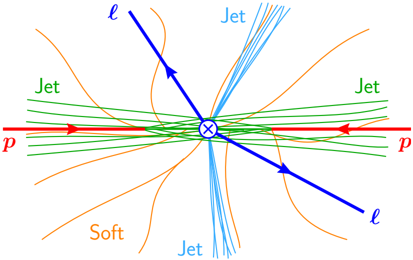
Resumming Jet Cross Sections in SCET

Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]

Effective theory of QCD describing what happens before/after hard interaction

Soft	Low-energy particles without preferred direction
Collinear	Energetic jets along incoming and outgoing directions



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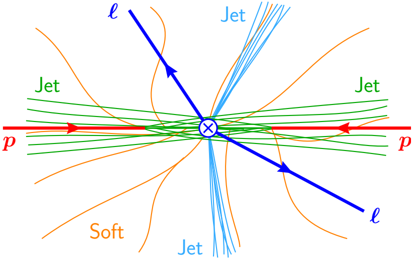
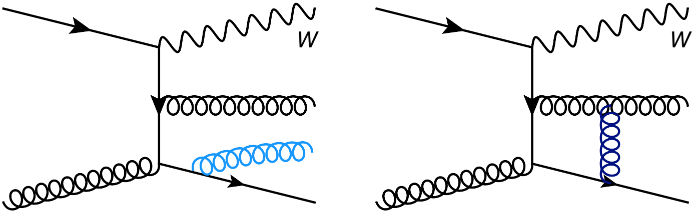


Advantages of SCET

- Systematic power counting, expansion in soft and collinear limits and gauge invariance manifest at the Lagrangian level
- Operator definitions of soft and collinear contributions
- "Nonsingular" corrections can be included systematically

Example: $pp \to W + 2$ jets



QCD

- Real and virtual corrections
- IR divergences cancel after costly phase-space integration

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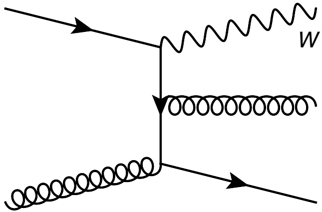

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SCET

- Match QCD onto SCET: partons correspond to energetic jets

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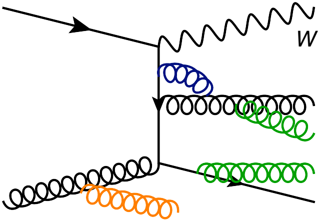


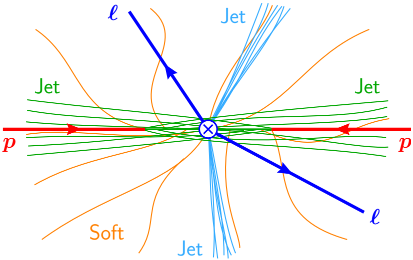
QCD

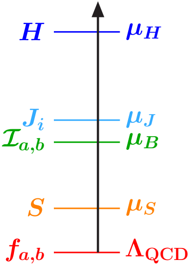
- Real and virtual corrections
- IR divergences cancel after costly phase-space integration

SCET

- Match QCD onto SCET: partons correspond to energetic jets
- Only (IR finite part of) virtual QCD corrections
- Real radiation described by collinear and soft degrees of freedom

Factorization for Exclusive Jet Cross Sections





Contributions at different energy scales:

$$\mathbf{d}\boldsymbol{\sigma} = \text{hard interaction} \otimes \text{PDFs} \otimes \text{ISR} \otimes \text{FSR} \otimes \text{soft radiation}$$

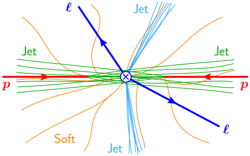
$$\mathbf{d}\boldsymbol{\sigma} = H_N \times \Big[(f_{a,b} \otimes \mathcal{I}_{a,b}) \times \prod_{j=1}^{N} J_j \Big] \otimes S_N$$

SCET allows us to *derive* factorized cross section

- Each function has a precise definition in the effective field theory
- RG evolution between scales resums logarithms of ratios of scales

SCET Ingredients

$$\mathrm{d}\sigma_N = H_N \times \left[B_a \times B_b \times \prod_{j=1}^{N} J_j \right] \otimes S_N$$



Beam, jet, soft functions:

- Contain virtual and integral over real radiation in collinear or soft limit (each function is separately IR finite)
- Depend on jet definition/observable
- Beam and jet function depend on parton type and energy
- Soft function depends on color representation and direction of *all* partons

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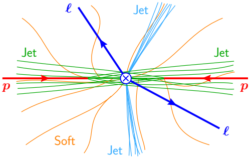

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Hard function

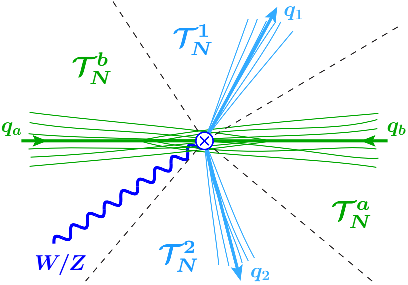
- Contains hard virtual corrections
- Independent of observable and precise form of factorization theorem
- Depends on process and hard kinematics (hard = away from singular limits)

N-Jettiness Event Shape

[Stewart, FT, Waalewijn]

$$\mathcal{T}_N = \sum_k \min\Big\{\frac{2q_a\cdot p_k}{Q_a}, \frac{2q_b\cdot p_k}{Q_b}, \frac{2q_1\cdot p_k}{Q_1}, \frac{2q_2\cdot p_k}{Q_2}, \dots, \frac{2q_N\cdot p_k}{Q_N}\Big\}$$
$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$

- $q_{a,b}$, q_j: light-like reference directions from overall minimization (or other jet algorithm like anti-k_T)
- $Q_{a,b}$, Q_j: determine distance measure of particle k to beam and jet directions
- Divides phase space into N jet regions and 2 beam regions



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[Stewart, FT, Waalewijn]

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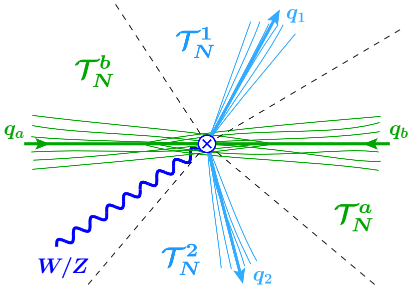
- $Q_{a,b}$, Q_j: determine distance measure of particle k to beam and jet directions
- Divides phase space into N jet regions and 2 beam regions
- Jet mass $m_i^2 = Q_i \mathcal{T}_N^i$

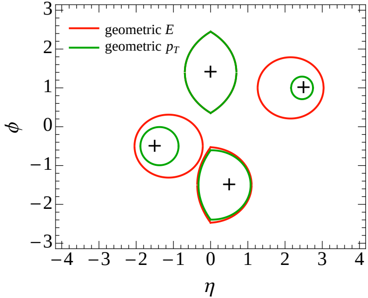


For small $\mathcal{T}_N \ll Q$ final state contains exactly N jets ($+2$ ISR jets)
(Generalization of thrust for $e^+e^- \to 2$ jets to $pp \to N$ jets)

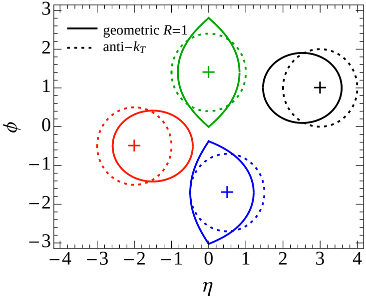
N-Jettiness Jets

$$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a\cdot p_k}{Q_a}, \frac{2q_b\cdot p_k}{Q_b}, \frac{2q_1\cdot p_k}{Q_1}, \frac{2q_2\cdot p_k}{Q_2}, \dots, \frac{2q_N\cdot p_k}{Q_N} \right\}$$

$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$



$Q_j = E_j$ or $Q_j = p_{Tj}$



$Q_j = \rho(R, \eta_j) E_j$ to have same jet area as anti-k_T

- Can change this to make jet area circular as in anti-k_T [Thaler, Tilburg]

Resummation for N-Jettiness

N-jettiness is theoretically ideal exclusive N-jet algorithm

- Factorization and resummation is known analytically [Stewart, Tackmann, WW]

$$\frac{\mathrm{d}\sigma_N}{\mathrm{d}\mathcal{T}_N} = H_N \times \left[B_a \times B_b \times \prod_{j=1}^{N} J_j \right] \otimes S_N$$

- Ingredients for NNLL+NLO resummation are available:
  - Inclusive quark and gluon jet functions are known even to NNLO [Becher, Neubert; Becher, Bell]
  - Inclusive quark and gluon beam functions [Fleming, Leibovich, Mehen; Stewart, Tackmann, WW; Berger et al.]
  - N-jettiness soft function [Jouttenus, Stewart, Tackmann, WW]
  - 3-loop cusp and 2-loop non-cusp anomalous dimensions [Moch, Vermaseren, Vogt; Aybat, Dixon, Sterman; Gardi, Magnea; Becher, Neubert]
  - Extract Hard NLO corrections from helicity amplitudes → *this talk*
- Progress in resummation with standard jet algorithms [Banfi, Salam, Zanderighi; Becher, Neubert;Tackmann, Walsh, Zuberi; Liu, Petriello]

Matching with Helicity Amplitudes

Matching QCD onto SCET

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SCET}} + \sum_k C_k \times O_k$$

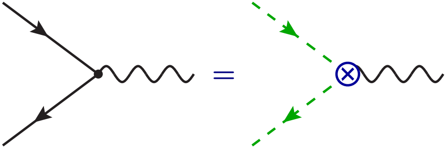
$$\mathcal{A}_{\text{QCD}} \stackrel{!}{=} \mathcal{A}_{\text{SCET}} = \sum_k \mathrm{i}C_k \times \langle O_k \rangle_{\text{SCET}}$$

Wilson coefficients

- Follow from matching amplitudes in full and effective theory
- Independent of IR regulator
- Depend on renormalization scheme (we use dim. reg. with $\overline{\text{MS}}$)
- Determine the hard function $H = CC^\dagger$

Schematic Matching At Tree Level

Consider e.g. Drell-Yan

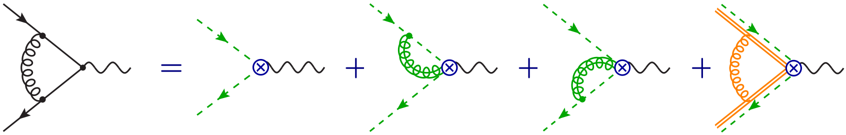


$$\mathcal{A}^{(0)}_{\text{QCD}} \stackrel{!}{=} \mathcal{A}^{(0)}_{\text{SCET}}$$

$$= \mathrm{i}C^{(0)}_{q\bar{q}} \times \langle O_{q\bar{q}} \rangle^{(0)}_{\text{SCET}} = \mathrm{i}C^{(0)}_{q\bar{q}}$$

where we normalize $O_{q\bar{q}}$ such that $\langle O_{q\bar{q}} \rangle^{(0)}_{\text{SCET}} \equiv 1$

Schematic Matching At One Loop



$$\mathcal{A}^{(1)} = \mathrm{i}C^{(1)}_{q\bar{q}} + \mathrm{i}C^{(0)}_{q\bar{q}} \times \langle O_{q\bar{q}} \rangle^{(1)}$$

$$\mathcal{A}^{(0)}\Big[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\Big] + \mathcal{A}^{(1)}_{\text{fin}} = \mathrm{i}C^{(1)}_{q\bar{q}} + \mathcal{A}^{(0)} \times \Big[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\Big]$$

Using dimensional regularization for both UV and IR

- Bare $\langle O_{q\bar{q}} \rangle^{(1)} = 0 \sim 1/\epsilon_{\text{IR}} - 1/\epsilon_{\text{UV}}$ vanishes (scaleless integrals)
- Adding UV counter-term $\delta_O \sim 1/\epsilon_{\text{UV}}$ leaves IR divergences
- IR divergences in SCET and QCD are equal
- Works to all orders: $\mathrm{i}C_{q\bar{q}} = \mathcal{A}_{\text{fin}}$

Building Blocks of SCET Operators

- Collinear quark field $\chi_{n,\omega}$ and gluon field $\mathcal{B}^{\mu}_{n\perp,\omega}$
- Contains collinear Wilson lines for gauge invariance, e.g. $\chi_n = W_n^\dagger \xi_n$
- Fields have fixed large momentum carried by their labels $\tilde{p}^\mu = \omega\, n^\mu/2$
- Sum over operators includes an integral over labels

$$\sum_k C_k\, O_k \quad \to \quad \int \prod_i \mathrm{d}\tilde{p}_i\, C_k(\{\tilde{p}_i\})\, O_k(\{\tilde{p}_i\})$$

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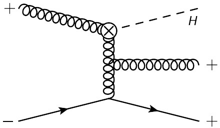
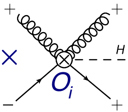
$\mathcal{A}(1^+ 2^+ 3_q^+ 4_{\bar{q}}^- 5_H) =$  $= \sum_i C_i \times$ 

- Textbook approach to spin unnecessary complicated:

$$\begin{aligned}
O_1 &= \bar{\chi}_{n_3,\omega_3}\, \not{n}_2\, \chi_{n_4,\omega_4}\, \mathcal{B}_{n_1\perp,\omega_1} \cdot \mathcal{B}_{n_2\perp,\omega_2}\, H_5 \\
O_2 &= \bar{\chi}_{n_3,\omega_3}\, \not{\mathcal{B}}_{n_1\perp,\omega_1}\, \chi_{n_4,\omega_4}\, n_4 \cdot \mathcal{B}_{n_2\perp,\omega_2}\, H_5 \\
O_3 &= \bar{\chi}_{n_3,\omega_3}\, \not{n}_1 \not{n}_2\, \not{\mathcal{B}}_{n_1\perp,\omega_1}\, \chi_{n_4,\omega_4}\, n_4 \cdot \mathcal{B}^{\perp}_{n_2,\omega_2}\, H_5 \\
&\dots
\end{aligned}$$

[Marcantonini, Stewart]

Helicity Fields

- Use standard spinor representation for polarization vectors

$$\varepsilon_+^\mu(p,k) = \frac{\langle p+|\gamma^\mu|k+\rangle}{\sqrt{2}\langle kp\rangle} \qquad \varepsilon_-^\mu(p,k) = -\frac{\langle p-|\gamma^\mu|k-\rangle}{\sqrt{2}[kp]}$$

- Define collinear gluon field and $q\bar{q}$-current of definite helicity

$$\mathcal{B}^a_{i\pm} = -\varepsilon_{\mp\mu}(n_i,\bar{n}_i)\,\mathcal{B}^{a\mu}_{n_i,\omega_i}$$

$$J^{\alpha\beta}_{ij\pm} = \mp\,\varepsilon^\mu_\mp(\tilde{p}_i,\tilde{p}_j)\,\frac{\bar{\chi}^\alpha_{n_i,-\omega_i\pm}\gamma_\mu\chi^\beta_{n_j,\omega_j\pm}}{\sqrt{2}\langle\tilde{p}_j\mp|\tilde{p}_i\pm\rangle}$$

- Resulting tree-level Feynman rules

$$\big\langle g^a_\pm(p)\big|\mathcal{B}^b_{i\pm}\big|0\big\rangle = \delta^{ab}\,\tilde{\delta}(\tilde{p}_i-p) \qquad \big\langle g^a_\mp(p)\big|\mathcal{B}^b_{i\pm}\big|0\big\rangle = 0$$

$$\big\langle q^{\alpha_1}_\pm(p_1)\,\bar{q}^{\alpha_2}_\mp(p_2)\big|J^{\beta_1\beta_2}_{12\pm}\big|0\big\rangle = \delta^{\alpha_1\beta_1}\,\delta^{\alpha_2\beta_2}\,\tilde{\delta}(\tilde{p}_1-p_1)\,\tilde{\delta}(\tilde{p}_2-p_2)$$

Helicity Operator Basis

Assemble helicity fields into helicity operators for each helicity configuration (S is symmetry factor for identical particles)

$$O^{a_1 a_2 \cdots \alpha_{i-1} \alpha_i \cdots \alpha_{n-1} \alpha_n}_{\pm\pm\cdots(\pm\cdots\pm)}(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_{i-1}, \tilde{p}_i, \ldots, \tilde{p}_{n-1}, \tilde{p}_n)$$
$$= S\, \mathcal{B}^{a_1}_{1\pm}\, \mathcal{B}^{a_2}_{2\pm} \cdots J^{\alpha_{i-1}\alpha_i}_{i-1,i\pm} \cdots J^{\alpha_{n-1}\alpha_n}_{n-1,n\pm}$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SCET}} + \sum_{\substack{\text{helicity}\\ \text{configurations}}} \int \prod_{i=1}^{n} \mathrm{d}\tilde{p}_i\, C^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(\tilde{p}_1, \ldots, \tilde{p}_n)\, O^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(\tilde{p}_1, \ldots, \tilde{p}_n)$$

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Assemble helicity fields into helicity operators for each helicity configuration (S is symmetry factor for identical particles)

$$O^{a_1 a_2 \cdots \alpha_{i-1}\alpha_i \cdots \alpha_{n-1}\alpha_n}_{\pm\pm\cdots(\pm\cdots\pm)}(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_{i-1}, \tilde{p}_i, \ldots, \tilde{p}_{n-1}, \tilde{p}_n) = S\, \mathcal{B}^{a_1}_{1\pm}\, \mathcal{B}^{a_2}_{2\pm} \cdots J^{\alpha_{i-1}\alpha_i}_{i-1,i\pm} \cdots J^{\alpha_{n-1}\alpha_n}_{n-1,n\pm}$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SCET}} + \sum_{\substack{\text{helicity}\\ \text{configurations}}} \int \prod_{i=1}^{n} \mathrm{d}\tilde{p}_i\, C^{a_1 \cdots \alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(\tilde{p}_1, \ldots, \tilde{p}_n)\, O^{a_1 \cdots \alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(\tilde{p}_1, \ldots, \tilde{p}_n)$$

Tree-level SCET amplitude projects out a single Wilson coefficient for given helicity configuration

$$\left\langle g^{a_1}_{+}(p_1) g^{a_2}_{-}(p_2) \cdots q^{\alpha_{n-1}}_{-}(p_{n-1}) \bar{q}^{\alpha_n}_{+}(p_n) \middle| \mathcal{L}_{\text{eff}} \middle| 0 \right\rangle^{(0)}_{\text{SCET}} = C^{a_1 a_2 \cdots \alpha_{n-1}\alpha_n}_{+-\cdot\cdot(\cdot\cdot-)}(p_1, p_2, \ldots, p_{n-1}, p_n)$$

Color Decomposition

Pick a complete basis of color-singlet structures to decompose coefficients

$$C_{+-\cdot\cdot(\cdot\cdot-)}^{a_1 a_2 \cdots \alpha_{n-1} \alpha_n} = \vec{T}^{\dagger\, a_1 a_2 \cdots \alpha_{n-1} \alpha_n} \cdot \vec{C}_{+-\cdot\cdot(\cdot\cdot-)}$$

e.g.

$$\vec{T}^{\dagger\,\alpha\beta} = \left(\delta^{\alpha\beta}\right) \qquad \vec{T}^{\dagger\,ab} = \left(\delta^{ab}\right)$$

$$\vec{T}^{\dagger\,a\alpha\beta} = \left(T^a_{\alpha\beta}\right) \qquad \vec{T}^{\dagger\,abc} = \left(if^{abc}, d^{abc}\right)$$

$$\vec{T}^{\dagger\,\alpha\beta\gamma\delta} = \left(\delta_{\alpha\delta}\,\delta_{\gamma\beta}\,,\,\delta_{\alpha\beta}\,\delta_{\gamma\delta}\right)$$

$$\vec{T}^{\dagger\,ab\alpha\beta} = \Big((T^a T^b)_{\alpha\beta}\,,\,(T^b T^a)_{\alpha\beta}\,,\,\mathrm{tr}[T^a T^b]\,\delta_{\alpha\beta}\Big) \qquad \text{etc ...}$$

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e.g.

$$\vec{T}^{\dagger\, \alpha\beta} = \left(\delta^{\alpha\beta}\right) \qquad \vec{T}^{\dagger\, ab} = \left(\delta^{ab}\right)$$

$$\vec{T}^{\dagger\, a\alpha\beta} = \left(T^a_{\alpha\beta}\right) \qquad \vec{T}^{\dagger\, abc} = \left(i f^{abc}, d^{abc}\right)$$

$$\vec{T}^{\dagger\, \alpha\beta\gamma\delta} = \left(\delta_{\alpha\delta}\, \delta_{\gamma\beta}\,,\ \delta_{\alpha\beta}\, \delta_{\gamma\delta}\right)$$

$$\vec{T}^{\dagger\, ab\alpha\beta} = \Big((T^a T^b)_{\alpha\beta}\,,\ (T^b T^a)_{\alpha\beta}\,,\ \mathrm{tr}[T^a T^b]\, \delta_{\alpha\beta} \Big) \qquad \text{etc ...}$$

Using the same basis as in the color decomposition of the amplitudes

$$\mathcal{A}_{\mathrm{QCD}}(g_+ g_- \cdots q_- \bar{q}_+) = \sum_k \vec{T}_k^{\dagger\, a_1 a_2 \cdots \alpha_{n-1} \alpha_n} A^k(1^+, 2^-, \ldots, n_{\bar{q}}^+)$$

$\overline{\mathrm{MS}}$ Wilson coefficients are equal to the color-ordered amplitudes to all orders

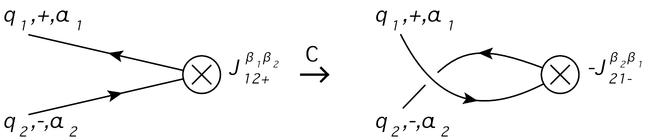
$$\vec{C}^k_{+-\cdot\cdot(\cdot\cdot-)}(p_1, p_2, \ldots, p_{n-1}, p_n) = A^k_{\mathrm{fin}}(1^+, 2^-, \ldots, n_{\bar{q}}^+)$$

C and P

- C and P for helicity fields

$$\mathrm{P}\,\mathcal{B}^{a}_{i\pm}(p_i)\,\mathrm{P} = \mathcal{B}^{a}_{i\mp}(p_i^{\mathrm{P}}) \qquad \mathrm{C}\,\mathcal{B}^{a}_{i\pm}\,T^{a}_{\alpha\beta}\,\mathrm{C} = -\mathcal{B}^{a}_{i\pm}\,T^{a}_{\beta\alpha}$$

$$\mathrm{P}\,J^{\alpha\beta}_{ij\pm}(p_i, p_j)\,\mathrm{P} = J^{\alpha\beta}_{ij\mp}(p_i^{\mathrm{P}}, p_j^{\mathrm{P}}) \qquad \mathrm{C}\,J^{\alpha\beta}_{ij\pm}\,\mathrm{C} = -J^{\beta\alpha}_{ji\mp}$$



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- C and P for helicity fields

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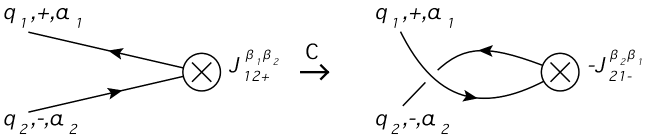

- C and P of operators easily determined

- For example for $O^{ab\,\alpha\beta}_{++(+)} = \frac{1}{2}\,\mathcal{B}^{a}_{1+}\,\mathcal{B}^{b}_{2+}\,J^{\alpha\beta}_{34+}$
  - Parity: $C_{++(+)} = C_{--(-)}$ up to a phase
    since Lorentz invariants $s_{ij} = (p_i + p_j)^2 = (p_i^{\mathrm{P}} + p_j^{\mathrm{P}})^2$
  - Charge conjugation relates $C_{++(+)}$ and $C_{++(-)}$:

$$\mathrm{C}\,O^{ab\,\alpha\beta}_{++(+)}(p_1, p_2; p_3, p_4)\,\mathrm{C} = -O^{ba\,\alpha\beta}_{++(-)}(p_1, p_2; p_4, p_3)$$

Examples

$gg q\bar{q}$: Basis and Matching

- Six helicity operators:

$$O^{ab\,\alpha\beta}_{++(+)} = \frac{1}{2}\,\mathcal{B}^a_{1+}\,\mathcal{B}^b_{2+}\,J^{\alpha\beta}_{34+}$$

$$O^{ab\,\alpha\beta}_{+-(+)} = \mathcal{B}^a_{1+}\,\mathcal{B}^b_{2-}\,J^{\alpha\beta}_{34+}$$

$$\dots$$

- Color structures:

$$\vec{T}^{\dagger ab\,\alpha\beta} = \big(T^a T^b \quad T^b T^a \quad \mathrm{tr}[T^a T^b]\mathbf{1}\big)_{\alpha\beta}$$

- C and P: only $C_{+\pm(+)}$ independent

$gg q\bar{q}$: Basis and Matching

- Six helicity operators:

$$O^{ab\,\alpha\beta}_{++(+)} = \frac{1}{2}\,\mathcal{B}^a_{1+}\,\mathcal{B}^b_{2+}\,J^{\alpha\beta}_{34+}$$

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$$\ldots$$

- Color structures:

$$\vec{T}^{\dagger ab\,\alpha\beta} = \Big(T^aT^b \quad T^bT^a \quad \mathrm{tr}[T^aT^b]1\Big)_{\alpha\beta}$$

- C and P: only $C_{+\pm(+)}$ independent

- QCD color decomposition:

$$\mathcal{A}(1 2\, 3_q^+ 4_{\bar{q}}^-) = \mathrm{i} \sum_{\sigma\in S_2} \big[T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}\big]_{\alpha_3\alpha_4} A(\sigma(1),\sigma(2);3_q^+,4_{\bar{q}}^-)$$
$$+\,\mathrm{i}\,\mathrm{tr}[T^{a_1}T^{a_2}]\,\delta_{\alpha_3\alpha_4}\,B(1,2;3_q^+,4_{\bar{q}}^-)$$

- Matching coefficients:

$$\vec{C}_{+-(+)}(p_1,p_2;p_3,p_4) = \begin{pmatrix} A_{\mathrm{fin}}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \\ A_{\mathrm{fin}}(2^-,1^+;3_q^+,4_{\bar{q}}^-) \\ B_{\mathrm{fin}}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \end{pmatrix}$$

and similarly for $\vec{C}_{++(+)}$

$gg q\bar{q}$: Matching Results

- Nonvanishing tree-level helicity amplitudes

$$A^{(0)}(1^+,2^-;3_q^+,4_{\bar q}^-)=-2g^2\,\frac{\langle 23\rangle\langle 24\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle}=2g^2\,\frac{\sqrt{|s_{13}\,s_{14}|}}{s_{12}}\,e^{i\Phi_{+-}}$$

$$A^{(0)}(2^-,1^+;3_q^+,4_{\bar q}^-)=-2g^2\,\frac{\langle 23\rangle\langle 24\rangle^3}{\langle 21\rangle\langle 14\rangle\langle 43\rangle\langle 32\rangle}=2g^2\,\frac{s_{13}\sqrt{|s_{13}\,s_{14}|}}{s_{12}\,s_{14}}\,e^{i\Phi_{+-}}$$

$$s_{ij}=(p_i+p_j)^2=2p_i\cdot p_j\,,\qquad e^{i\Phi_{+-}}=\frac{\langle 24\rangle}{[24]}\,\frac{[13][14]}{\sqrt{|s_{13}\,s_{14}|}}$$

  - Pull out convention-dependent overall phase

$gg q\bar{q}$: Matching Results

- Nonvanishing tree-level helicity amplitudes

$$A^{(0)}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = -2g^2 \frac{\langle 23\rangle\langle 24\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle} = 2g^2 \frac{\sqrt{|s_{13}\, s_{14}|}}{s_{12}}\, e^{i\Phi_{+-}}$$

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$$s_{ij} = (p_i+p_j)^2 = 2p_i\cdot p_j\,, \qquad e^{i\Phi_{+-}} = \frac{\langle 24\rangle}{[24]}\frac{[13][14]}{\sqrt{|s_{13}\, s_{14}|}}$$

  - Pull out convention-dependent overall phase
- One-loop helicity amplitudes were calculated by [Kunszt, Signer, Trocsanyi]

$$A^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)}\frac{\alpha_s}{4\pi}\Big[-\frac{2}{\epsilon^2}(C_A+C_F) + \frac{1}{\epsilon}(2C_F\, L_{12} + 2C_A\, L_{13} - 3C_F - \beta_0)\Big]\,,$$

$$A^{(1)}_{\rm fin}(1^+,2^-;3_q^+,4_{\bar{q}}^-) = A^{(0)}\frac{\alpha_s}{4\pi}\Big\{C_A\Big(-L_{13}^2 + L_{12/13}^2 + 1 + \frac{7\pi^2}{6}\Big) + C_F\Big(-L_{12}^2 + 3L_{12} - 8 + \frac{\pi^2}{6}\Big) + (C_A - C_F)\frac{s_{12}}{s_{14}}(L_{12/13}^2 + \pi^2)\Big\}$$

$$L_{ij} = \ln\Big(-\frac{s_{ij}}{\mu^2} - i0\Big) \qquad L_{ij/kl} = L_{ij} - L_{kl}$$

  - Cross check: IR divergences equal in QCD and SCET

$$\frac{\alpha_s}{4\pi}\Big[-\frac{2}{\epsilon^2}(C_A+C_F) + \frac{1}{\epsilon}\Big(-\beta_0 - 3C_F + 2\hat{\Delta}_{gg\,q\bar{q}}(\mu^2)\Big)\Big]\vec{C}^{(0)}_{+-(+)} = \begin{pmatrix} A^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \\ A^{(1)}_{\rm div}(2^-,1^+;3_q^+,4_{\bar{q}}^-) \\ B^{(1)}_{\rm div}(1^+,2^-;3_q^+,4_{\bar{q}}^-) \end{pmatrix}$$

$\hat{\Delta}_{gg\,q\bar{q}}$ = anomalous dim. mixing matrix

$gg q\bar{q}$: Hard Function

$$\mathrm{d}\sigma_2 = \int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}\Phi_2 \underbrace{\sum_{\kappa}}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_2^{\kappa} \hat{S}_2^{\kappa}]}_{\text{color trace}} \otimes [B_{\kappa_a} B_{\kappa_b} J_{\kappa_1} J_{\kappa_2}]$$

$$\hat{H}_2^{gg q\bar{q}} = \sum_{\lambda_1,\lambda_2,\lambda_3} \vec{C}_{\lambda_1\lambda_2(\lambda_3)} \vec{C}^{\dagger}_{\lambda_1\lambda_2(\lambda_3)}$$

- Calculate: $\vec{C}_{++(+)}$ and $\vec{C}_{+-(+)}$
- Identical particles: $\vec{C}_{-+(+)}(p_1,p_2,p_3,p_4) = \hat{V}\vec{C}_{+-(+)}(p_2,p_1,p_3,p_4)$
- Charge conjug.: $\vec{C}_{++(-)}(p_1,p_2,p_3,p_4) = -\hat{V}\vec{C}_{++(+)}(p_1,p_2,p_4,p_3)$
- Parity gives remaining $\vec{C}_{--(+)} = \vec{C}_{++(-)}$ etc.

$$\hat{V} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(interchanging color structures)}$$

$gg q\bar{q}$: Soft Function

$$d\sigma_2 = \int dx_a dx_b \int d\Phi_2 \underbrace{\sum_\kappa}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_2^\kappa \hat{S}_2^\kappa]}_{\text{color trace}} \otimes [B_{\kappa_a} B_{\kappa_b} J_{\kappa_1} J_{\kappa_2}]$$

- Soft function $\hat{S}_N^\kappa$ is matrix in color space
- At tree-level, soft function has no emissions,

$$\hat{S}_N{}^{b_1\cdots\beta_N a_1\cdots\alpha_N} \propto \mathbf{1} = \delta^{b_1 a_1} \cdots \delta^{\beta_N \alpha_N} = \sum_{a_1,\dots,\alpha_N} \vec{T}^{a_1\cdots\alpha_N} \vec{T}^{\dagger a_1\cdots\alpha_N}$$

E.g. for $gg q\bar{q}$

$$\mathbf{1}_{gg\,q\bar{q}} = \frac{C_A C_F}{2} \begin{pmatrix} 2C_F & 2C_F - C_A & 1 \\ 2C_F - C_A & 2C_F & 1 \\ 1 & 1 & C_A \end{pmatrix}$$

$gg q\bar{q}$: Soft Function

$$d\sigma_2 = \int dx_a dx_b \int d\Phi_2 \underbrace{\sum_{\kappa}}_{\text{parton types}} \underbrace{\mathrm{tr}[\hat{H}_2^{\kappa} \hat{S}_2^{\kappa}]}_{\text{color trace}} \otimes [B_{\kappa_a} B_{\kappa_b} J_{\kappa_1} J_{\kappa_2}]$$

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- At tree-level, soft function has no emissions,

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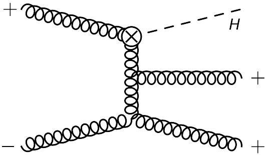
E.g. for $gg q\bar{q}$

$$1_{gg\, q\bar{q}} = \frac{C_A C_F}{2} \begin{pmatrix} 2C_F & 2C_F - C_A & 1 \\ 2C_F - C_A & 2C_F & 1 \\ 1 & 1 & C_A \end{pmatrix}$$

- The N-jet soft function at NLO:
  - Jet angularities of cone-jets [Ellis, Hornig, Lee, Vermilion, Walsh]
  - N-Jettiness [Jouttenus, Stewart, Tackmann, WW]

$ggggH$: Basis



- Five helicity operators:

$$O^{abcd}_{++++} = \frac{1}{4!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4+}H_5$$

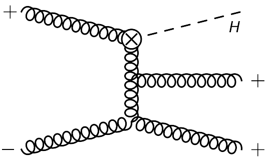
$$O^{abcd}_{+++-} = \frac{1}{3!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4-}H_5$$

$$\dots$$

- Six color structures:

$$\vec{T}^{\dagger abcd} = \Big(\tfrac{1}{2}(\mathrm{tr}[abcd] + \mathrm{tr}[dcba]), \dots \\ \mathrm{tr}[ab]\,\mathrm{tr}[cd],\ \dots\Big)$$

$ggggH$: Basis



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$$\dots$$

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$$\vec{T}^{\dagger abcd} = \Big(\tfrac{1}{2}(\mathrm{tr}[abcd] + \mathrm{tr}[dcba]), \dots \; \mathrm{tr}[ab]\,\mathrm{tr}[cd], \; \dots\Big)$$

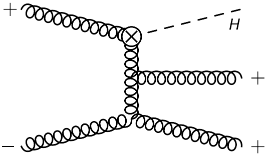
- Parity: $\vec{C}_{++++} = \vec{C}_{----}$ and $\vec{C}_{+++-} = \vec{C}_{---+}$ up to a phase
- Under charge conjugation

$$\mathrm{C}\, O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\, \vec{T}^{abcd}\, \mathrm{C} = O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\, \vec{T}^{dcba}$$

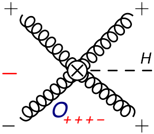
$\mathrm{tr}[abcd] - \mathrm{tr}[dcba]$ is thus not allowed

$gggg H$: Basis

$$\mathcal{A}(1^+2^+3^+4^-5_H) =$$



$$= iC_{+++-}$$



- Five helicity operators:

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$$\dots$$

- Six color structures:

$$\vec{T}^{\dagger abcd} = \left(\tfrac{1}{2}(\mathrm{tr}[abcd] + \mathrm{tr}[dcba]), \dots \right.$$
$$\left. \mathrm{tr}[ab]\,\mathrm{tr}[cd],\ \dots\right)$$

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- Under charge conjugation

$$\mathrm{C}\, O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\vec{T}^{abcd}\,\mathrm{C} = O^{abcd}_{\lambda_1\lambda_2\lambda_3\lambda_4}\vec{T}^{dcba}$$

$\mathrm{tr}[abcd] - \mathrm{tr}[dcba]$ is thus not allowed

$ggggH$: Intrinsic phases

- QCD partial amplitudes

$$\mathcal{A}(1234) = \mathrm{i} \sum_{\sigma \in S_4/Z_4} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)} a_{\sigma(3)} a_{\sigma(4)}] A\big(\sigma(1), \sigma(2), \sigma(3), \sigma(4)\big)$$
$$+ \mathrm{i} \sum_{\sigma \in S_4/Z_2^3} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)}] \mathrm{tr}[a_{\sigma(3)} a_{\sigma(4)}] B\big(\sigma(1), \sigma(2), \sigma(3), \sigma(4)\big)$$

- Matching coefficients are

$$\vec{C}_{++--}(p_1, p_2, p_3, p_4) = \begin{pmatrix} 2A_{\mathrm{fin}}(1^+, 2^+, 3^-, 4^-) \\ \vdots \\ B_{\mathrm{fin}}(1^+, 2^+, 3^-, 4^-) \\ \vdots \end{pmatrix}$$

$gggH$: Intrinsic phases

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$$+ \mathrm{i} \sum_{\sigma \in S_4/Z_2^3} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)}] \mathrm{tr}[a_{\sigma(3)} a_{\sigma(4)}] B\big(\sigma(1),\sigma(2),\sigma(3),\sigma(4)\big)$$

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- Tree-level helicity amplitudes calculated by [Kauffmann, Desai, Risal]

$$A^{(0)}(1^+,2^+,3^-,4^-;5_H) = -2\left[\frac{[12]^4}{[12][23][34][41]} + \frac{\langle 34\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\right]$$

$ggggH$: Intrinsic phases

- QCD partial amplitudes

$$\mathcal{A}(1234) = \mathrm{i} \sum_{\sigma \in S_4/Z_4} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)} a_{\sigma(3)} a_{\sigma(4)}] A\big(\sigma(1),\sigma(2),\sigma(3),\sigma(4)\big)$$
$$+\mathrm{i} \sum_{\sigma \in S_4/Z_2^3} \mathrm{tr}[a_{\sigma(1)} a_{\sigma(2)}]\mathrm{tr}[a_{\sigma(3)} a_{\sigma(4)}] B\big(\sigma(1),\sigma(2),\sigma(3),\sigma(4)\big)$$

- Matching coefficients are

$$\vec{C}_{++--}(p_1,p_2,p_3,p_4) = \begin{pmatrix} 2A_{\mathrm{fin}}(1^+,2^+,3^-,4^-) \\ \vdots \\ B_{\mathrm{fin}}(1^+,2^+,3^-,4^-) \\ \vdots \end{pmatrix}$$

- Tree-level helicity amplitudes calculated by [Kauffmann, Desai, Risal]

$$A^{(0)}(1^+,2^+,3^-,4^-;5_H) = 2\left[\frac{s_{12}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}} + e^{-2\mathrm{i}\phi_2}\frac{s_{34}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}}\right] e^{\mathrm{i}\Phi}$$

- Two independent intrinsic phases:

$$e^{\mathrm{i}\phi_1} = \frac{\langle 13\rangle\langle 24\rangle}{\langle 12\rangle\langle 34\rangle}\frac{\sqrt{|s_{12}s_{34}|}}{\sqrt{|s_{13}s_{24}|}} \qquad e^{\mathrm{i}\phi_2} = \frac{\langle 14\rangle\langle 23\rangle}{\langle 12\rangle\langle 34\rangle}\frac{\sqrt{|s_{12}s_{34}|}}{\sqrt{|s_{14}s_{23}|}}$$

  - Independent of phase conventions
  - ϕ_i can be written in terms of s_{ij} and $\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$

$pp \to Hj$: Basis and Matching

- Eight helicity operators:

$$O^{abc}_{+++} = \frac{1}{3!}\,\mathcal{B}^a_{1+}\,\mathcal{B}^b_{2+}\,\mathcal{B}^c_{3+}\,H_4$$

$$\ldots$$

$$O^{a\,\alpha\beta}_{+(+)} = \mathcal{B}^a_{1+}\,J^{\alpha\beta}_{23+}\,H_4$$

$$\ldots$$

- Color structures:

$$\vec{T}^{\dagger abc} = (\mathrm{i} f^{abc})\,,\, \vec{T}^{\dagger a\,\alpha\beta} = (T^a_{\alpha\beta})$$

- C and P: only $C_{++\pm}$, $C_{+(+)}$ indep.

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- Matching coefficients:

$$\begin{aligned}\vec{C}_{+++}(p_1, p_2; p_3, p_4) &= \big(A_{\text{fin}}(1^+, 2^+; 3^+, 4_H)\big) \\ &= \Big(\frac{1}{\sqrt{2}} \frac{m_H^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} + \mathcal{O}(\alpha_s)\Big)\end{aligned}$$

$$\ldots$$

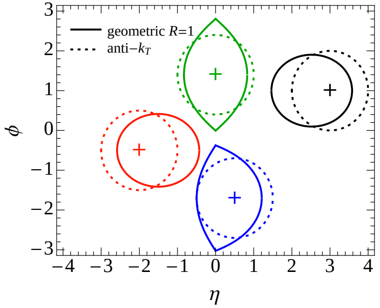
- NLO matching coefficients known [Schmidt]

$pp \to Hj$: Jet Mass Spectrum

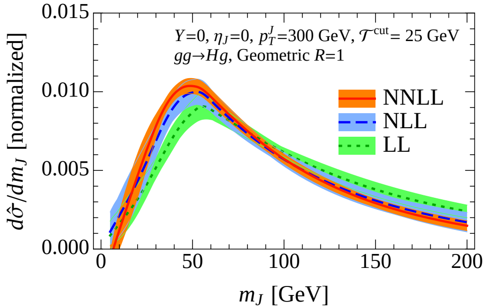
- Calculate jet mass of N-jettiness jets [Jouttenus, Stewart, Tackmann, WW]

$$\frac{d\sigma^{H+1j}}{dm_J} = \int dx_a dx_b \int d\Phi_2 \underbrace{\sum_\kappa}_{\text{parton types}} \hat{H}_2^\kappa \hat{S}_2^\kappa \otimes [B_{\kappa_a} B_{\kappa_b} J_{\kappa_1}]$$

Shape of jet region:

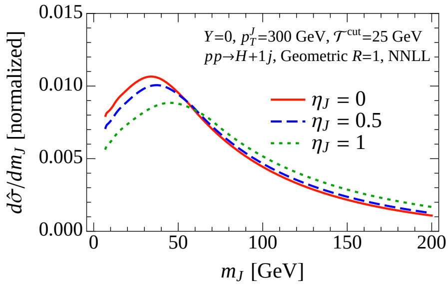


Good perturbative convergence:

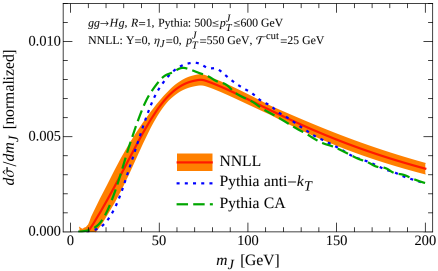


$pp \to Hj$: Jet Mass Spectrum

Study kinematics:



Comparison with Pythia:



- Working on $pp \to Zj$ and $pp \to jj$ as well

Conclusions

Tools to combine generic NLO with NNLL resummation are available

- Natural next step beyond NLO + LL parton showers
- This is precisely what SCET is designed to do
- At NNLL the choice of jet definition is important $\to$ N-jettiness

We introduce a helicity operator basis such that

$$\mathcal{A}_{\text{fin}}^{\text{QCD}}(1^{+}2\cdots n_{\bar{q}}^{+}) = \mathrm{i}C^{a_1\cdots\alpha_n}_{+\cdot\cdot(\cdot\cdot-)}(p_1,\ldots,p_n)$$

- Individual color-ordered amplitudes are needed, not just their sum
- Using resummation, virtual amplitudes can be used to get physical cross sections without expensive integrations over real emissions

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Thank you